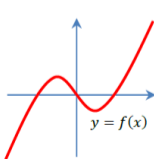


# Symmetry properties

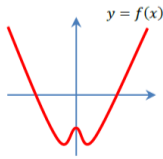
## Definition 1.34

A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is called

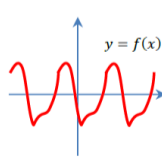
- (i) an **odd function** if  $f(-x) = -f(x)$  for every  $x \in \mathbb{R}$ ;
- (ii) an **even function** if  $f(-x) = f(x)$  for every  $x \in \mathbb{R}$ ;
- (iii) a **periodic function** if there exists  $c > 0$  such that  $f(x + c) = f(x)$  for every  $x \in \mathbb{R}$ .  $c$  is called a **period** of  $f$ . If there is a smallest such positive number  $c$ , then it is called the **fundamental period** of  $f$ .



$f: \mathbb{R} \rightarrow \mathbb{R}$  is odd

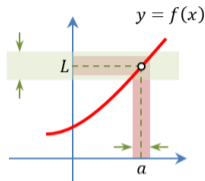


$f: \mathbb{R} \rightarrow \mathbb{R}$  is even



$f: \mathbb{R} \rightarrow \mathbb{R}$  is periodic

# Informal definition of limit



## Definition 2.1 (Finite limit at a number)

Let  $a$  be a real number and  $f$  be a function which is well-defined on an open interval that contains  $a$ , except possibly at  $a$ . A real number  $L$  is called a “**limit of  $f(x)$  as  $x$  tends to  $a$** ” if  $f(x)$  can be made *as close to  $L$  as possible*, provided that  $x$  gets *sufficiently close* to  $a$  **and  $x \neq a$** . In symbols we write either

$$f(x) \rightarrow L \text{ as } x \rightarrow a \quad \text{or} \quad \lim_{x \rightarrow a} f(x) = L.$$