

DP Mathematics (Level 1)

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The Center for the Development of the Gifted and Talented,
The Hong Kong University of Science and Technology



Course introduction

September 7, 2024

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We will study behaviors of different functions in various aspects.

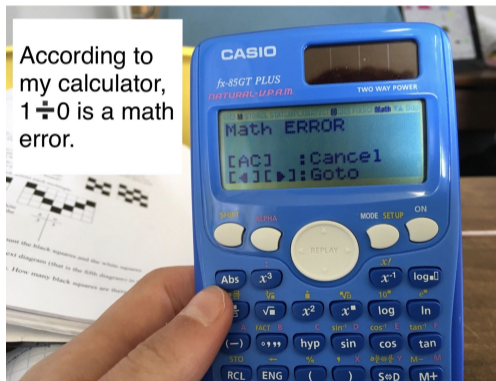
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What is actually happening here?

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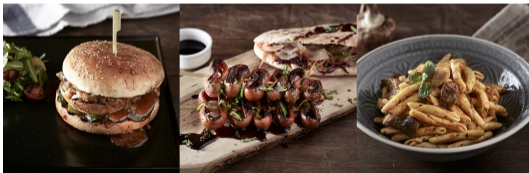
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- Without using a calculator, how do you find a solution to a **complicated equation** such as

$$x = \cos x?$$

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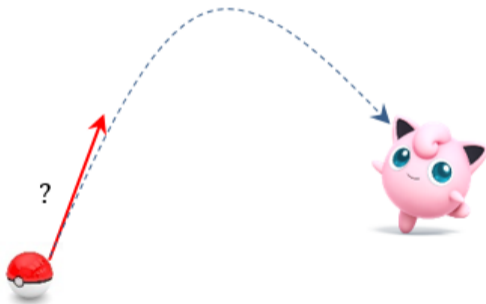
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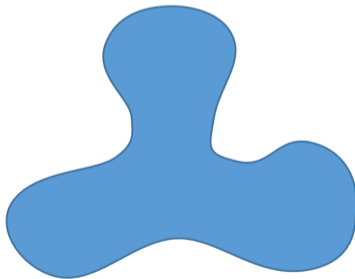
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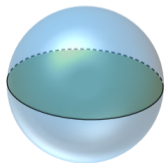
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- Why is the **area** of a circular disk of radius r given by

$$A = \pi r^2?$$

Why is the **volume** of a spherical ball of radius r given by

$$V = \frac{4\pi}{3}r^3?$$

How do we compute the **volume**, **surface area**, **length**, etc. of other geometric shapes?



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- Some **proof-based problems** will be included.

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The main purpose of this test to better allocate students to a Mathematics course (**Level 1** vs. **Pre-stage**) (**English** vs. **Cantonese**) that is **more suitable** for them.

- You do not need to study or prepare particularly for this test.
- Usage of calculators will not be allowed (and will not be necessary).