Chapter 1: Introduction to graph theory

Dr. Chu Kwun Lun, Alan

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Reference

- P.K. Sun, Lecture Note in GFQR1045 Making a Smart Decision, Department of Mathematics, Hong Kong Baptist University, 2020.
- W.C. Shiu and P.K. Sun, A First Course in Graph Theory, Department of Mathematics, Hong Kong Baptist University, 2014.
- J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Macmillan, 1976
- J. Clark and D.A. Holton, A First Look at Graph Theory, World Scientific, 1996.

What is a Graph?

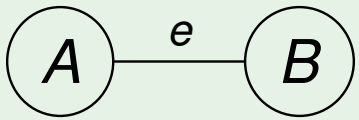
Definition 1

A *graph* (or sometimes called *multigraph*) is a combination of

- a nonempty set of *vertices*, and
- a set of *edges* (can possibly be empty)

so that each edge are connected to two vertices. These vertices are said to be *adjacent* to each other.

Example

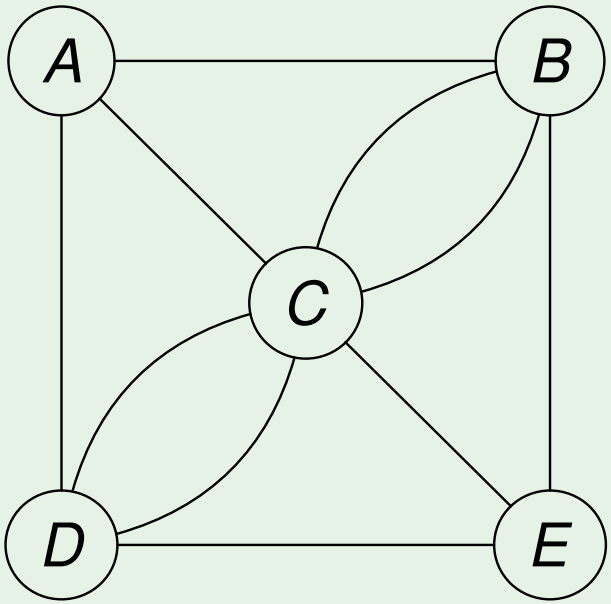


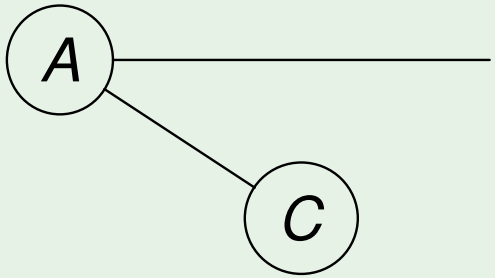
Vertices A and B are adjacent. Edge e is *incident* to A and B.

What is a Graph?

Example

The figure on the left is a graph, while the one on the right is not.

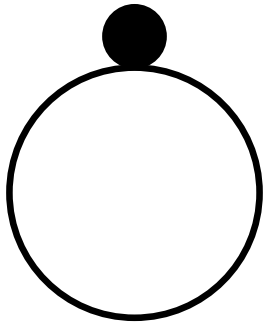




Loops and Parallel Edges

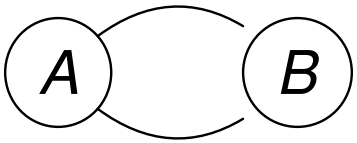
Definition 2

A *loop* consists of one vertex and one edge where the edge is connecting the vertex itself at two ends.



Definition 3

If two or more edges are connected between the same pair of vertices, then those edges are called *parallel edges* or *multiple edges*.



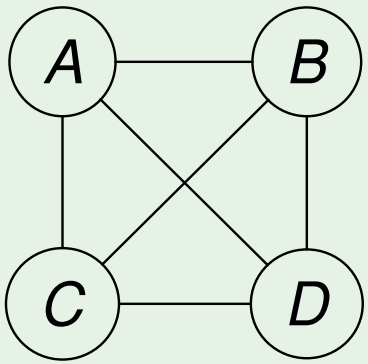
Simple Graphs

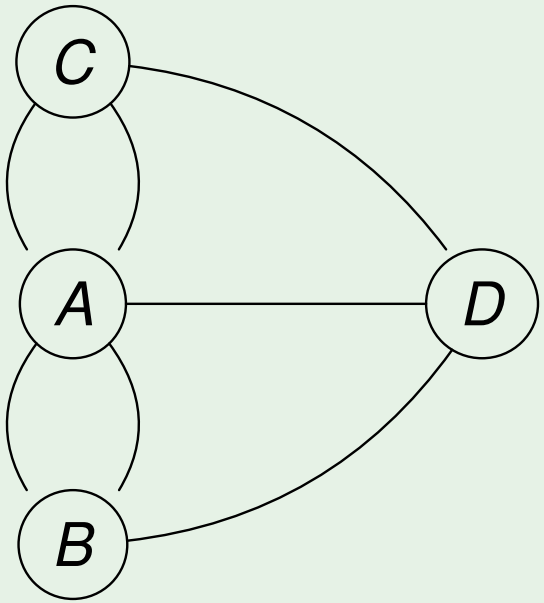
Definition 4

A graph is a *simple graph* if it has no loop nor parallel edges.

Example

The left is a simple graph, while the right is a multigraph.





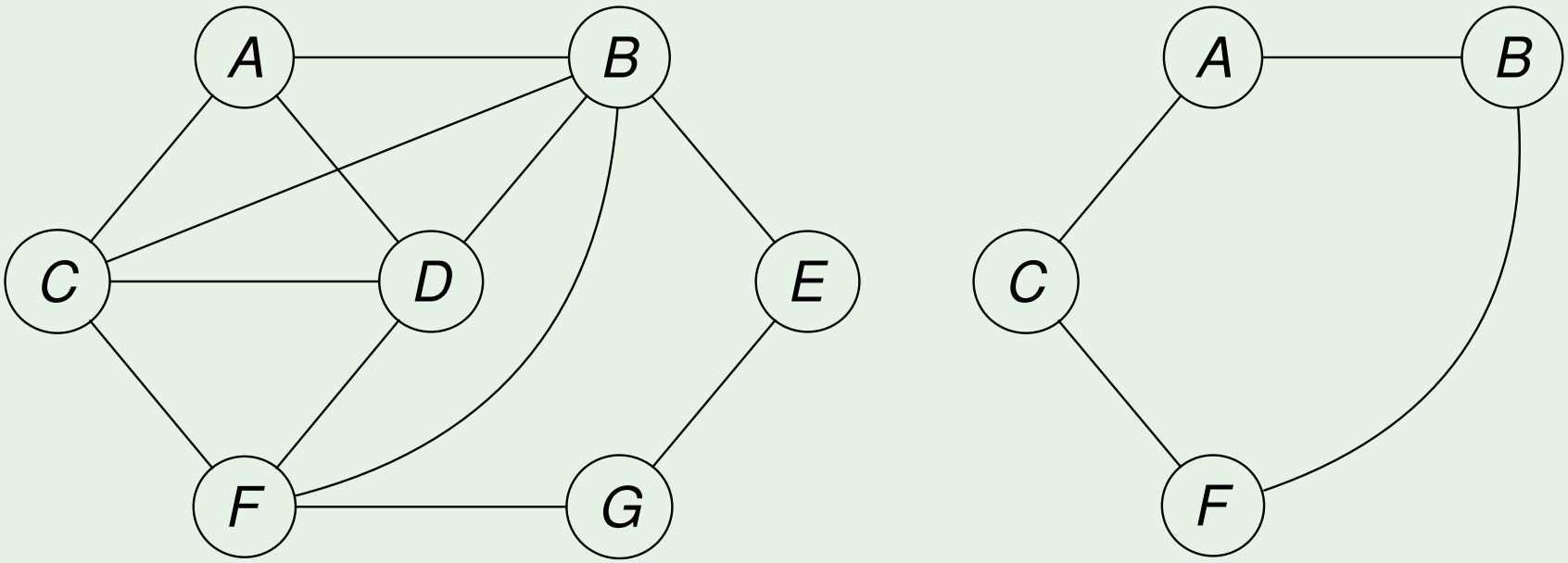
Subgraphs

Definition 5

Let G be a graph. If a subset of vertices and a subset of edges of G form a graph, then it is called a *subgraph* of G.

Example

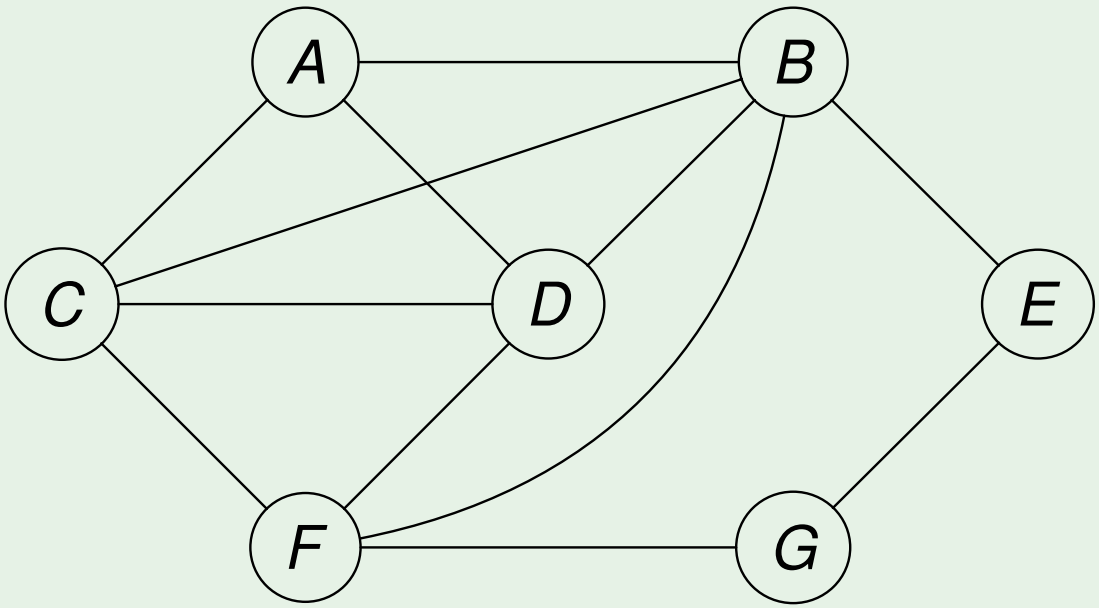
The graph on the right is a subgraph of the graph on the left.

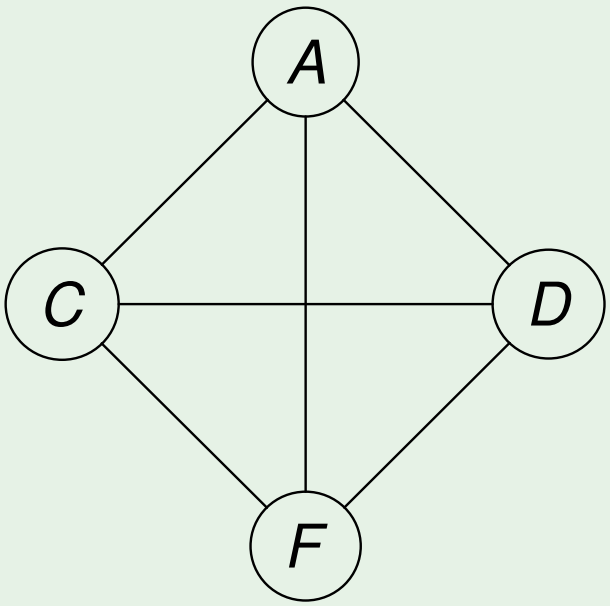


Subgraphs

Example

The graph on the right is **NOT** a subgraph of the graph on the left





since the edge between A and F on the right is not in the original graph.

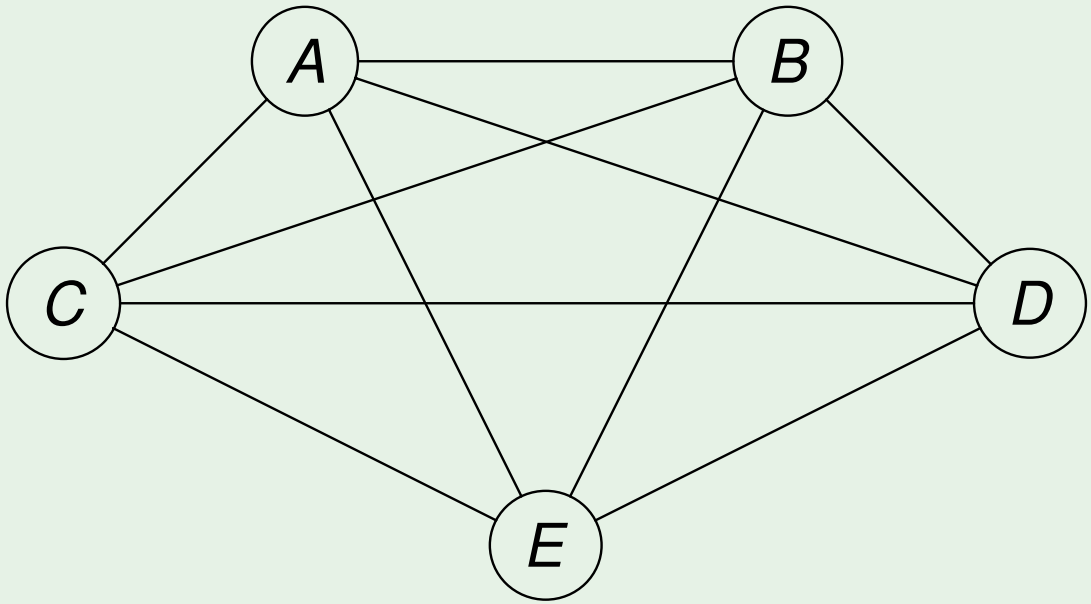
Complete Graphs

Definition 6

A *complete graph* (denoted by K_n) is a simple graph with n vertices such that any two vertices are adjacent.

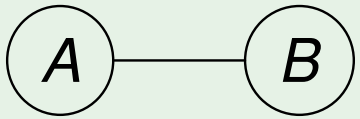
Example

The following graph is a K_5.

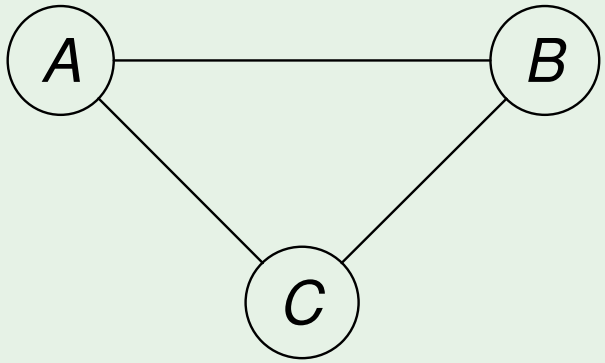


Example of Complete Graphs

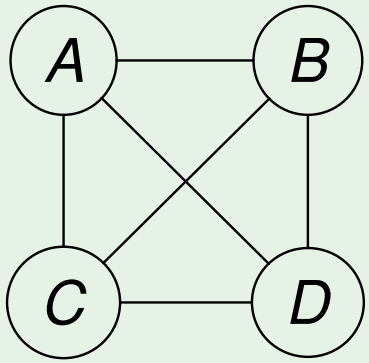
Example



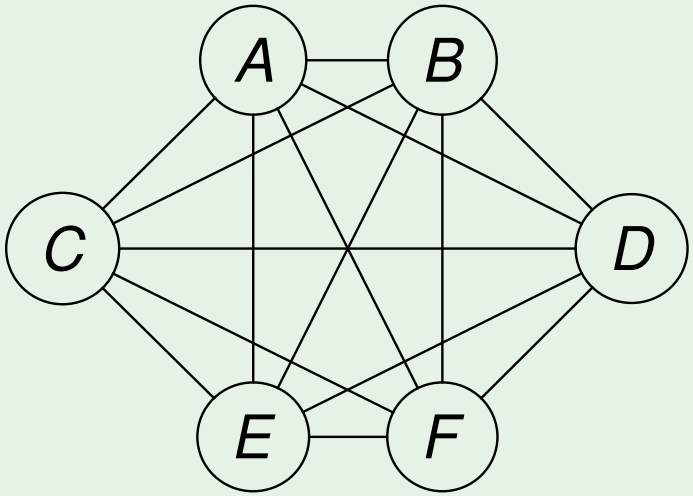
K_2



K_3



K_4



K_6